Alternative Investment Criteria

Second Criterion: Internal Rate of Return (IRR)

IRR is the discount rate (K) at which the present value of benefits are just equal to the present value of costs for the particular project

$$\sum_{i=0}^{t} \frac{B_t - C_t}{(1 + K)^t} = 0$$

Note: IRR is a mathematical concept, not an economic or financial criterion

Common uses of IRR:

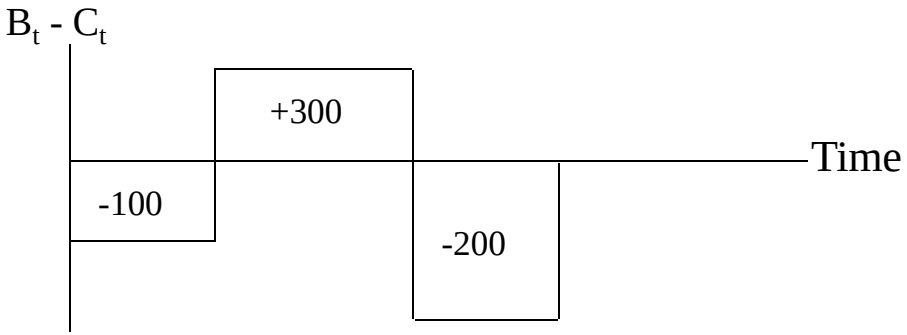
(a) If the IRR is larger than the cost of funds then the project should be undertaken

(b) Often the IRR is used to rank mutually exclusive projects. The highest IRR project should be chosen

An advantage of the IRR is that it only uses information from the project

Difficulties with the Internal Rate of Return Criterion

First difficulty: Multiple rates of return for project



Solution 1: K = 100%; NPV= -100 + 300/(1+1) + -200/(1+1)^2 = 0

Solution 2: K = 0%; NPV= -100+300/(1+0)+-200/(1+0)^2 = 0

What is NPV at K = 50% or 150%?

Difficulties with the Internal Rate of Return Criterion *(contd)*

Second difficulty: Projects of different sizes and also strict alternatives

Year	0	1	2	3	...	...	ã
Project A	-2,000	+600	+600	+600	+600	+600	+600
Project B	-20,000	+4,000	+4,000	+4,000	+4,000	+4,000	+4,000

NPV and IRR provide different conclusions:

Opportunity cost of funds = 10% and the revenues of 600 and 2000 are assumed to be occurring in perpetuity (for ever), NPV is 600 and 2000 divided by discount rate.

NPV of A : 600/0.10 - 2,000 = 6,000 - 2,000 = 4,000

NPV of B : 4,000/0.10 - 20,000 = 40,000 - 20,000 = 20,000

Hence, NPV of B > NPV of A but IRR of A is higher:

IRR_A : $600/K_A - 2,000 = 0$ or $K_A = 0.30$

IRR_B : $4,000/K_B - 20,000 = 0$ or $K_B = 0.20$

Hence, $K_A > K_B$

Difficulties with the Internal Rate of Return Criterion *(contd)*

Third difficulty: Projects of different lengths of life and strict alternatives

Opportunity cost of funds = 8%

Project A: Investment costs = 1,000 in year 0

Benefits = 3,200 in year 5

Project B: Investment costs = 1,000 in year 0

Benefits = 5,200 in year 10

NPV A : $-1{,}000 + 3{,}200/(1.08)^5 = 1{,}177.86$

NPV B : $-1{,}000 + 5{,}200/(1.08)^{10} = 1{,}408.60$

Hence, NPV of B > NPV of A

IRR_A : $-1{,}000 + 3{,}200/(1+K_A)^5 = 0$ which implies that $K_A = 0.262$

IRR_B : $-1{,}000 + 5{,}200/(1+K_B)^{10} = 0$ which implies that $K_B = 0.179$

Hence, $K_A > K_B$

Difficulties with the Internal Rate of Return Criterion *(contd)*

Fourth difficulty: Same project but started at different times

Project A: Investment costs = 1,000 in year 0

Benefits = 1,500 in year 1

Project B: Investment costs = 1,000 in year 5

Benefits = 1,600 in year 6

NPV_A : $-1{,}000 + 1{,}500/(1.08) = 388.88$

NPV_B : $-1{,}000/(1.08)^5 + 1{,}600/(1.08)^6 = 327.68$

Hence, NPV A > NPV B

IRR_A : $-1{,}000 + 1{,}500/(1+K_A) = 0$ which implies that $K_A = 0.5$

IRR_B : $-1{,}000/(1+K_B)^5 + 1{,}600/(1+K_B)^6 = 0$ which implies that $K_B = 0.6$

Hence, $K_B > K_A$

When is IRR valid?

- IRR can be used to compare investments when they have the same:
  - Scale/size
  - Timing
  - Term/length
  - Pattern of benefits
- For example, can compare term deposits of same term, or return on equity shares over same period
- Rate of return is a useful summary measure of the efficiency of a unit of investment over a unit period, but has limitations in seeking maximum NPV in optimizing a project or selecting between alternatives.

Alternative Investment Criteria

Third Criterion: Benefit-Cost Ratio

Benefit-Cost Ratio (R) = Present Value Benefits/Present Value Costs

Basic rule:

If benefit-cost ratio (R) >1, then the project should be undertaken.

Problems?

Sometimes it is not possible to rank projects with the Benefit-Cost Ratio

- Mutually exclusive projects of different sizes
- Mutually exclusive projects and recurrent or operating costs subtracted out of benefits and costs versus benefits reported gross of operating costs
- Not necessarily true that $R_A > R_B$ that project "A" is better

Benefit-Cost Ratio *(contd)*

First Problem: The Benefit-Cost Ratio does not adjust for mutually exclusive projects of different sizes. **For example:**

Project A: PV^0 of Costs = \$5.0 M, PV^0 of Benefits = \$7.0 M

NPV_A = \$2.0 M **$R_A = 7/5 = 1.4$**

Project B: PV^0 of Costs = \$20.0 M, PV^0 of Benefits = \$24.0 M

NPV_B = \$4.0 M $R_B = 24/20 = 1.2$

According to the Benefit-Cost Ratio criterion, project A should be chosen over project B because $R_A > R_B$, but the NPV of project B is greater than the NPV of project A. So, *project B should be chosen*

Second Problem: The Benefit-Cost Ratio does not adjust for mutually exclusive projects with recurrent or operating costs subtracted out of benefits and costs versus benefits reported gross of operating costs. **For example:**

Project A: Total Costs = \$5.0 M Recurrent Costs = \$1.0 M

(i.e. Fixed Costs = \$4.0 M) PV^0 of Gross Benefits= \$7.0 M

$R_A = (7-1)/(5-1) = 6/4 = 1.5$ *(compared to 1.4 above)*

Project B: Total Costs = \$20.0 M Recurrent Costs = \$18.0 M

(i.e. Fixed Costs = \$2.0 M) PV^0 of Gross Benefits= \$24.0 M

$R_B = (24-18)/(20-18) = 6/2 = 3$ *(compared to 1.2 above)*

Hence, project B should be chosen over project A under Benefit-Cost Criterion.

Conclusion:

The Benefit-Cost Ratio CANNOT be used to rank projects